Sample Problems For Final

Instructions: The test consists of 15 multiple choice questions (worth 4 pts each) and 6 problems (worth 10 points each).

Part I - Limit Problems (4 pts each)

1) $\lim_{x\to 0} \frac{Sin(2x)}{3x} =$

(a) 3/2 (b) 2/3 (c) 0 (d) 1 (e) does not exit

2) $lim_{x\to\infty} \frac{-2x^2+7x-3}{(1-x)(x+3)} =$

(a) -2 (b) -3 (c) 2 (d) 3 (e) none of these

3) $\lim_{h\to 0} \frac{\sqrt{4+h}-2}{h} =$

(a) 2 (b) 1/2 (c) 1/4 (d) 1 (e) ∞

4) $lim_{x\to\pi}\, sin(x + sin(x)) =$

(a) π (b) -1 (c) 1 (d) 0 (e) $\cos(\pi)\,(1-\cos(\pi))$

5) $\lim_{h\to 0} \frac{tan^{-1}(e^h)-\pi/4}{h} =$

(a) 1 (b) 1/2 (c) 0 (d) $-\pi/4$ (e) ∞

Part II - Derivative Problems (4 pts each)

6) $f(x) = \frac{x^2+2}{x^4-3x^2+1}$ $f(x) =$

(a) $\frac{(2x)(x^4-3x^2+1)+(x^2+2)(4x^3-6x)}{(x^4-3x^2+1)^2}$ (b) $\frac{(2x)(x^4-3x^2+1)-(x^2+2)(4x^3-6x)}{(x^4-3x^2+1)^2}$

(c) $\frac{1}{x^4-3x^2+1}$ (d) $\frac{(x^3+x-2)-(x^2+2)(3x^2+1)}{(x^4-3x^2+1)^2}$

(e) (2x) $\frac{1}{x^4-3x^2+1} + \frac{1}{x^4-3x^2+1}$

7) $g(x) = e^x(\tan x - \sin x)$ $g'(x) =$

(a) $e^x(sec^2x - \cos x)$ (b) $e^x(\tan x - \sin x) + e^x\left(\frac{1}{1+x^2} - \frac{1}{\sqrt{1-x^2}}\right)$

(c) $xe^{x-1}(\tan x - \sin x) + e^x(sec^2x - \cos x)$

(d) $e^x(\tan x - \sin x + sec^2x - \cos x)$

8) If y is implicitly defined by $e^{x/y} = x - y$ then $y' =$

(a) $\frac{e^{\frac{x}{y}}y-y^2}{e^{\frac{x}{y}}x-y^2}$ (b) $1 - e^{\frac{x}{y}}$ (c) $\frac{e^{\frac{x}{y}}xy-y^2}{e^{\frac{x}{y}}x-y^2}$ (d) $\frac{e^{\frac{x}{y}}y-y^2}{e^{\frac{x}{y}}-y^2}$

(e) $\frac{e^{\frac{x}{y}}y-y^2+y^2}{e^{\frac{x}{y}}x}$

9) $y = (\sin x)^x \qquad y' =$

(a) $\ln(\sin x) + x\frac{\cos x}{\sin x}$ (b) $(\sin x)^x \cos x$ (c) $x(\sin x)^{x-1}\cos x$

(d) $(\sin x)^x\left(\ln(\sin x) + x\frac{\cos x}{\sin x}\right)$ (e) $(\sin x)^x\left(\ln(\sin x) - x\frac{\cos x}{\sin x}\right)$

10) A car moves along the x axis with position from the starting point given by some function, $S(t)$. If the acceleration, $a(t) = S''(t)$ is positive, then the SPEED is

(a) Increasing (b) Decreasing (c) Can't tell without more information. i.e. may be increasing or decreasing.

Part III - Integration Problems (4 pts each)

11) Suppose $f(x)$ is continuous everywhere and

$$F(x) = \int_0^x f(t)dt$$

Which of the following is true. i.e. the statement is true for the reason given.

(a) $F(x)$ is differentiable everywhere because of the Mean Value Theorem

(b) $F(x)$ is continuous everywhere because $F(x)$ has a derivative by the Fundamental Theorem of Calculus and differentiable functions are continuous.

(c) $F(x)$ is continuous because the Intermediate Value Theorem proves that the integral of a continuous function is continuous.

(d) $F(x)$ has a derivative because the chain rule implies that the integral of a continuous function has a derivative.

(e) The function $F(x)$ does not have a derivative because it is an integral.

12) $\int \frac{x}{\sqrt{1-4x^2}}dx =$

(a) $-\frac{1}{4}\sqrt{1-4x^2} + C$ (b) $\frac{1}{12}(1-4x^2)^{-3/2} + C$ (c) $\frac{1}{4}(1-4x^2)^{-3/2} + C$

(d) $xsin^{-1}(2x) + C$ (d) $\sqrt{1-4x^2} + C$

13) $\int_1^e \frac{\ln x}{x} dx =$

(a) $e^2/2$ (b) 1 (c) 1/2 (d) 2 (e) $e - 1$

14) Compute $\int_0^1 \frac{3x}{\sqrt[3]{x^2+1}} dx$

(a) $\frac{9}{4}(\sqrt[3]{4} - 1)$ (b) $(\sqrt[3]{4} - 1)$ (c) $\frac{\sqrt[3]{25}}{4}$ (d) $\frac{9}{4}$ (e) $\frac{9}{4}(\sqrt[3]{25} - \sqrt[3]{4})$

15) $R_n = \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}}\right)$ is an nth Riemann sum on the interval [0, 1]. Then $\lim_{n\to\infty} R_n =$

(a) 1 (b) 1/2 (c) 3/2 (d) 2/3 (e) 2

Part IV - Non-Multiple Choice Problems (10 pts each)

16) Use implicit differentiation to find an equation of the tangent line to the curve defined implicitly by $\sin(x + y) = 2x - 2y$ at the point (π, π)

17) The angle of elevation of the sun is decreasing at a rate of 0.25 rad/hr. How fast is the shadow cast by a 400 ft tall building increasing when the angle of elevation of the sun is $\pi/6$?

18) If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

19) Suppose $f(0) = 0$, $f(\pi) = 0$, and $f''(x) = 2e^x + 3\sin x$. Find $f(x)$.

20) Using the definition of derivative, find $f'(x)$ where $f(x) = \sqrt{2 - 3x}$

21) Calculate $\int_{-1}^{2}(x^2 - 2x + 1)dx$ using Riemann sums.